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Search for chaotic features in the arrival times of air showers

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Abstract. – We study sequences of times between successive arrivals of air showers detected in the EAS-TOP experiment (primary energy between 70 and 1000 TeV) in order to establish their nature, whether stochastic or chaotic.

Introduction. – The sequences of times Δt_i between successive arrivals (TBSA) of air showers have so far been considered to be uniformly random. In the last International Cosmic Ray Conference, the Kinki and the Osaka Air Shower Groups have instead claimed that short sequences (number of events $128 \leq N \leq 512$) of TBSA of showers characterized by a certain number of consecutive small values of Δt_i may occasionally exhibit a low fractal dimension whose value varies between 1.3 and 4.5 [1]-[3]. If these results were confirmed, they could actually lead to many intriguing speculations: is the fractal dimension acquired as a consequence of the traversal of the solar cavity or instead originated in the galactic magnetic field? Which feature of the field causes the fractal dimension? Which mechanism is responsible for "translating" the local field feature into the fractal dimension of the air showers?

Our many years of involvement in the chaotic analysis of various cosmic-ray signals have made us extremely cautious (somebody would say even skeptical) about the interpretation of the fractal results in this field. Separation of the differentiable non-linear dynamics from stochasticity is definitely not an easy task, as proven by the analysis of the signals from the underground muon detector in Artyomovsk [4], [5] and of those from the electromagnetic detector of the EAS-TOP experiment [6]; only extensive numerical studies have allowed us to understand that the finite fractal dimension determined for these signals is a direct consequence of the coloured random noise properties of the time series (associated with power spectra which behave as $\sim f^{-\alpha}$) and has nothing to do with strange attractors. It thus seemed natural to

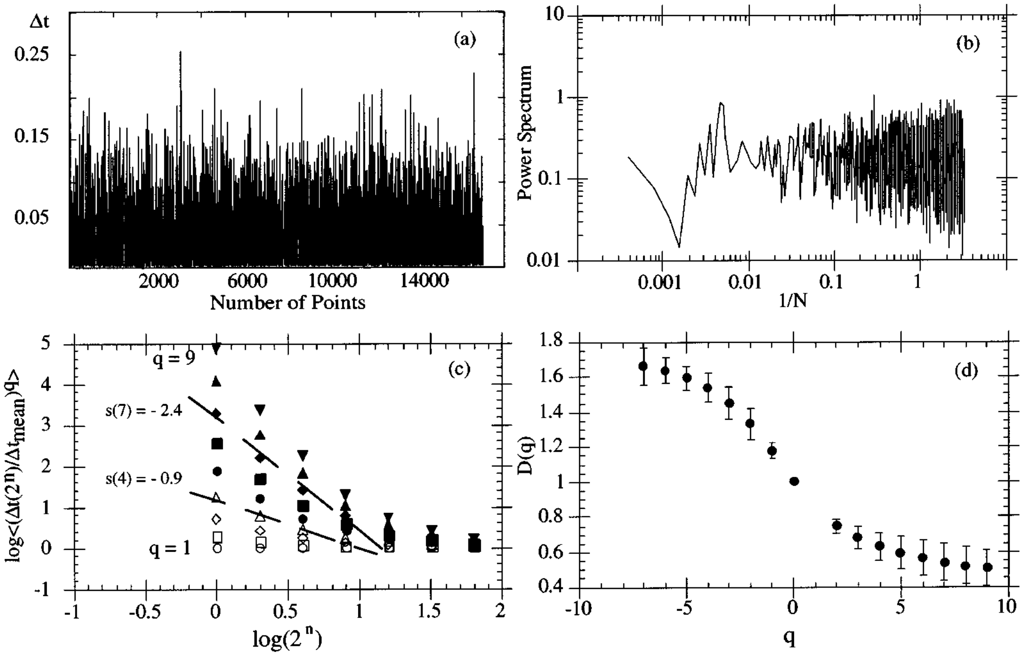

Fig. 1. – Example of a TBSA sequence recorded in the EAS-TOP experiment. *a*) Plot of Δt_i sequences. *b*) Power spectrum. *c*) Plots of $\log\langle(\Delta t(2^n)/\Delta t_{\text{mean}})^q\rangle$ *vs.* $\log(n)$ for $1 \le q \le 9$; also shown is the linear fitting procedure used to derive the slopes $s(q)$. *d*) Plot of the generalized dimension $D(q)$. (Note that $s(q) > s(q')$ and $D(q) > D(q')$, for $q' > q$.)

exploit our familiarity with the fractal and chaotic diagnostics to verify also the assumptions on the nature of the TBSA sequences (stochastic or chaotic?) using the large amount of air shower data available in the EAS-TOP experiment.

Analysis of the TBSA sequences from the EAS-TOP experiment. – We selected several TBSA sequences constituted by a number of events $10\,000 \le N \le 20\,000$, with four different values of primary energy: 70 TeV, 270 TeV, 500 TeV and 1000 TeV (corresponding to four Δt-distributions characterized by progressively larger average values). Figure 1 *a*) shows an example of such a sequence. We notice the presence of rapid fluctuations all along the sequence, detectable also in the shape of the power spectrum (the "white-noise" pattern of fig. 1 *b*)). The Fourier phases distribution, not shown here for brevity, is uniform over the interval $(0, 2\pi)$. The lack of a dominant frequency region does not allow the estimate of the evolution period of the system, which is the fundamental input parameter for the calculation of the maximum positive Lyapunov exponent. A complementary perspective on the behaviour of the measured sequences at different scales may be obtained by constructing a set of data whose elements are the averages of the original sequence over an increasing number of points 2^n, and by studying the scaling symmetry of the moments of order q of the set: $\langle(\Delta t(2^n)/\Delta t_{\text{mean}})^q\rangle \simeq n^{s(q)}$. The ensemble of the slopes $s(q)$, determined by a linear fitting procedure, is then used to obtain the so-called "generalized" dimension function $D(q)$. We see that, because of the rapid fluctuations present in all sequences, the process of averaging over an increasing number of points causes the plots of $\log\langle(\Delta t(2^n)/\Delta t_{\text{mean}})^q\rangle$ *vs.* $\log(n)$ to exhibit steeper and steeper slopes $s(q)$ as q increases (an example is shown in fig. 1 *c*) for $q > 0$). This behaviour translates into a plot

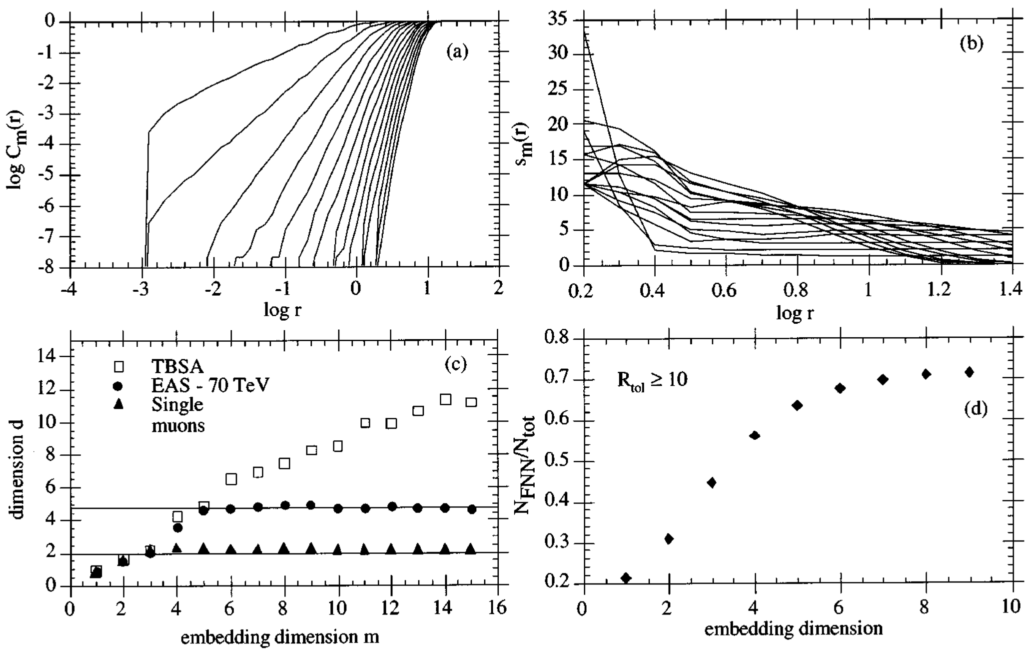

Fig. 2. – Example of the results of the Grassberger and Procaccia algorithm for the estimate of the correlation dimension of a TBSA sequence. *a*) $C_m(r)$ integrals; *b*) Slopes $s_m(r)$. *c*) Value of the slopes $s_m(r)$ in the range $0.5 \leq \log(r) \leq 0.8$ plotted as a function of m, compared with the results obtained in [6] for the single muon and EAS time series. *d*) Results of the false nearest-neighbours estimate of the correct embedding dimension.

$D(q)$ which satisfies the condition $D(q) > D(q')$ for $q' > q$ for the existence of structure on all scales (see fig. 1 *d*)).

The EAS-TOP data have been subjected to three of the most common analysis techniques from dynamical-systems theory: estimate of the Grassberger and Procaccia correlation integrals, false nearest-neighbours test and study of the self-similar properties. The complete references and the details on the methods used are reported in [4]-[8].

The Grassberger and Procaccia procedure for estimating the correlation dimension of measured data is based on the embedding of the original data in an m-dimensional pseudo-phase space and focus on the scaling properties of the m-dimensional integral correlation integrals $C_m(r)$. For all data investigated, irrespective of the primary energy considered, we obtain that the $C_m(r)$ integrals (an example is shown in fig. 2 *a*)) scale roughly as $\simeq r^m$, *e.g.*, their slopes $s_m(r)$, shown in fig. 2 *b*), show no signs of saturation to a particular correlation dimension d, but rather a clear asymptotic increase with the value of the embedding dimension. To better evidence this lack of convergence, the value of the slopes $s_m(r)$ in the range $0.5 \leq \log(r) \leq 0.8$ obtained from fig. 2 *b*) are plotted as a function of m in fig. 2 *c*) together with the results obtained in [6] for the single-muon time series (convergence to a correlation dimension $d = 2.2$) and the EAS time series (convergence to $d = 4.8$).

The information given by the correlation integrals is confirmed by the results of the "false nearest neighbours" test for the estimate of the correct embedding dimension. The basic idea of this method is that, given a system with an attractor, if we use an embedding dimension m which is too small to unfold the attractor, not all points that lie close to one another will be neighbours because of the dynamics. Some of these points will actually be far from each other and appear as neighbours because the geometric structure of the attractor has been projected down onto a smaller space. The percentage of false nearest neighbours $N_{\mathrm{FNN}}/N_{\mathrm{TOT}}$ which characterizes the passage from $(m-1)$ to m, plotted as a function of m, indicates the rate of convergence to the correct embedding dimension m^*, which sets a lower limit to the possible value of the correlation dimension. Figure 2 *d*) shows a typical result on the percentage of false nearest neighbours obtained for our TBSA sequences, with a clear asymptotic increase of $N_{\mathrm{FNN}}/N_{\mathrm{TOT}}$ with the value of m.

The analysis of the self-similar properties yields results in agreement with those derived with the Grassberger and Procaccia algorithm: the scaling exponent, estimated in two different and independent ways, is definitely equal to zero, *e.g.*, $d \to \infty$.

The original TBSA sequences have successively been fragmented in smaller and smaller pieces (down to $N = 100$ points) to be analysed separately; the results obtained in this way are completely equal to those obtained for the full data set. We are thus convinced that a stochastic "white-noise" interpretation of the TBSA sequences recorded in the EAS-TOP experiment is the natural conclusion of the results presented in this section, in complete agreement with the traditional picture.

Possible interpretation of the Kinki and Osaka results. – When contrasting the previous conclusion with the Kinki and Osaka results it is important to point out that the data from the EAS-TOP experiment show no evidence of the process of "clustering", *e.g.* the large number of consecutive small values of Δt_i, which the researchers from the Japanese installations say to characterize the sequences for which they find a low fractal dimension. In order to understand how this clustering may impact the determination of the correlation dimension we decided to reproduce it artificially by "doping" EAS-TOP sequences characterized by length N and average delay Δt with one or more subsets of length n^* and average delay $\Delta t^* < \Delta t$. We then proceeded to apply the Grassberger and Procaccia algorithm to the various sequences obtained by using different values for n^*/N and $\Delta t^*/\Delta t$. The results of this analysis show that the shape of the correlation integrals is very sensitive to the choice of the two parameters.

For very small values of the two parameters, such that the product of the two $\nu = (n^*/N)(\Delta t^*/\Delta t)$ is $< 10^{-4}$, the correlation integrals show a definite departure from the stochastic scaling $\simeq r^m$ in favour of a $\simeq r^0$ scaling over a wide range of r. This anomalous scaling is reminding of the $\simeq r$ scaling introduced by the procedure of filtering [9]; in both cases the resulting kink is a "reaction" of the algorithm to some abrupt changes in the sequence examined, in one case the introduction of a small subset with different features, in the other the cutting-off of the higher-frequency components. Only for values of the two parameters, such that ν is $> 10^{-2}$, the correlation integrals approach the stochastic scaling $\simeq r^m$ observed for the original EAS-TOP sequences. The general picture of the behaviour of the correlation integrals of sequences with $10^{-5} \le n \le 5 \cdot 10^{-2}$ is given in fig. 3 (for graphical simplicity we report only the results relative to the $\log C_{15}(r)$ curves). It is conceivable that the correlation integrals in their transition from one extreme scaling $\simeq r^{15}$ to the other extreme case $\simeq r^0$ may accidentally reproduce an r^d scaling $(1 < d < m)$ which, by itself, could easily be interpreted as the signature of a chaotic system with correlation dimension d. We feel that this interpretation could well explain not only the small fractal dimension determined by the Japanese researchers for the sequences with some degree of clustering, but also the variability of this value $(1.3 < d < 4.5)$, which otherwise would constitute another puzzle (unfortunately, we have no direct information on the actual structure of their sequences, since no examples are given in [1]-[3], and we cannot evaluate which range of n is involved). The basic problem encountered by an experimentalist who searches for low-dimensional chaos in a particular data sequence is the enormous number of deceiving factors which offer intriguing and appealing

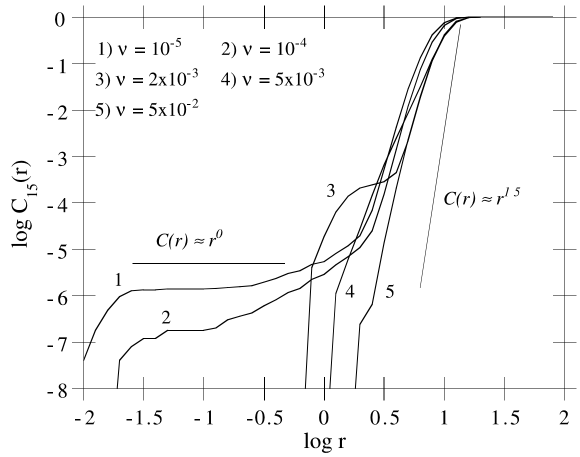

Fig. 3. – Correlation integrals $C_{15}(r)$ for five EAS-TOP sequences with length N and average delay Δt doped with one or more subsets of length n^* and average delay $\Delta t^* < \Delta t$; the parameter ν is the product $(n^*/N)(\Delta t^*/\Delta t)$. Also shown are the two extreme scaling behaviours $\simeq r^0$ and $\simeq r^{15}$.

suggestions. We know it even too well from direct experience: two of us were involved in the analysis of surface wave time series which consistently gave a value of the correlation dimension around 7 coupled with a positive largest Lyapunov exponent, a combination which seemed a very promising signature of non-linear dynamics until we proved that it was the consequence of accidental mimickings on the part of a classical stochastic Gaussian process [8].

To conclude our investigation we asked the question: if the EAS-TOP TBSA sequences were to contain a subset characterized by a true low fractal dimension (without any hypothesis on its origin) would we be able to detect it? To answer this question we doped again a few EAS-TOP sequences, this time using one or more subsets of computer-generated data with correlation dimension $d = 5$. We then proceeded to calculate the Grassberger and Procaccia correlation integrals of the various sequences. The results obtained showed a strong dependence on the fraction ϵ of data with $d = 5$ present in the sequence. We observe lack of convergence to any finite dimension for $\epsilon \leq 25\%$; we obtain $d \simeq 6.9$ for $\epsilon = 50\%$, $d = 5.3$ for $\epsilon = 75\%$, and finally $d = 5.0$ for $\epsilon = 90\%$. As a last test, we investigated the response of the Grassberger and Procaccia algorithm to small data samples (as those considered by the Kinki and Osaka groups). Using our computer-generated data with correlation dimension $d = 5$, we obtain $d = 4.8$ for $N = 500$ points, $d = 4.3$ for $N = 300$ and $d = 2.8$ for $N = 100$, confirming the well-known upper bound that the size of the data sets on the value of the dimension which can be determined.

Conclusions. – In the previous sections we have presented a study, from the perspective of dynamical-systems theory, of several sequences of times between successive arrivals of air showers detected in the EAS-TOP experiment. The primary energies considered a range between 70 and 1000 TeV. The motivation of our study has been to check the stochastic nature of the sequences, which has been recently questioned by some results from the Kinki and Osaka Air Shower groups.

The search for chaotic features in the TBSA sequences from the EAS-TOP experiment has been carried out using various independent approaches; the results of this multiple evaluation all agree in indicating that all the sequences are stochastic, and, in particular, have features similar to white noise. Out of $\simeq 75\,000$ air showers considered we find no candidate for chaotic behaviour. Our conclusion thus goes in the opposite direction relative to that reached by the researchers of the Japanese groups; this contradiction could be related to a possible difference in the structure of the sequences detected in the various experiments, and to the difficulty of reaching an unambiguous assessment of deterministic chaos in sequences characterized by a large number of consecutive small values of Δt_i.

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